PRANDTL NUMBER FOR LIQUIDS

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It is established on the basis of experimental data for water and toluene that: 1) the Prandtl number can be expressed in a form analogous to an equation of state; 2) there are linear relations between the Prandtl number and the transport properties.

The Prandtl number, which is a measure of the relative influence of thermal conductivity and viscosity, can be written

$$\mathrm{Pr} = \frac{\eta c_p}{\lambda}. \tag{1}$$

A check shows that the Prandtl number for liquids can be written in a form analogous to an equation of state. The validity of this assertion can be demonstrated for two liquids: ordinary water and toluene.

It was shown previously that the equation of state

$$\frac{pv}{RT} = 1 + B\rho + E\rho^4 \tag{2}$$

describes the specific volume of water over the temperature range from 0 to 350°C and over pressures from p_s to 1000 bars, within the experimental error [1], and it was shown that the equation of state

$$\frac{pv}{RT} = 1 + B\rho + H\rho^7 \tag{3}$$

describes the specific volume of toluene at temperatures from 25 to 300°C and at pressures from p_s to 500 bars [1, 2].

It has also been established that the Prandtl numbers for water and toluene are given by

$$\frac{\mathrm{Pr}}{\mathrm{Pr}'} = 1 + B_p\rho + E\rho^4 \tag{4}$$

and

$$\frac{\mathrm{Pr}}{\mathrm{Pr}'} = 1 + B_p\rho + H_p\rho^7. \tag{5}$$

With pv/RT = 1 and Pr/Pr' = 1, the coefficients of these equations should be related in the following manner:

$$-\left(\frac{B}{E}\right)_{p,\,v,\,T} = \rho_0^3. \tag{2'}$$

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$$-\left(\frac{B}{E}\right)_p = \rho_s^3, \tag{4'}$$

for water and

$$-\left(\frac{B}{H}\right)_{p,\,v,\,T} = \rho_0^6, \tag{3'}$$

$$-\left(\frac{B}{H}\right)_p = \rho_s^6 \tag{5'}$$

for toluene.

The validity of Eqs. (4) and (5) is confirmed by the linearity of the functional dependence of the combination $\mathrm{Pr}/\mathrm{Pr}' - 1/\rho$ on ρ^3 for water on seven isotherms at 50° steps from 0 to 300°C; the validity of these equations has also been confirmed for toluene by the linearity of the functional dependence of this combination on ρ^6 on nine isotherms at 25° steps from 25 to 250°C.

In the calculations, the numerical values of the Prandtl number for water are taken from [3], and those for toluene are calculated from Eq. (1) on the basis of our experimental data [4-7].

The values of the coefficients B_p, E_p, and B_p, H_p in Eqs. (4) and (5), which depend on the temperature, are calculated from the condition for a linear dependence of the combinations specified above on ρ^3 and ρ^6, respectively, and are approximated by the following polynomials: for water,

$$B_p = \sum_{i=0}^{6} b_i t^i \tag{6}$$

and

$$E_p = \sum_{i=0}^{6} e_i t^i, \tag{7}$$

where

$$\begin{aligned}
b_0 &= 171140\cdot 10^{-5}, & e_0 &= -171240\cdot 10^{-5},\\
b_1 &= -118454\cdot 10^{-7}, & e_1 &= 125844\cdot 10^{-7},\\
b_2 &= -497907\cdot 10^{-10}, & e_2 &= -124841\cdot 10^{-10},\\
b_3 &= 860300\cdot 10^{-12}, & e_3 &= 242000\cdot 10^{-13},\\
b_4 &= -391178\cdot 10^{-14}, & e_4 &= -181711\cdot 10^{-14},\\
b_5 &= 890933\cdot 10^{-17}, & e_5 &= 962133\cdot 10^{-17},\\
b_6 &= -796444\cdot 10^{-20}, & e_6 &= -180978\cdot 10^{-19};
\end{aligned}$$

for toluene,

$$B_p = \sum_{0}^{5} b_i t^i \tag{8}$$

and

$$H_p = \sum_{0}^{5} h_i t^i, \tag{9}$$

where

$$\begin{aligned}
b_0 &= -146560\cdot 10^{-5}, & e_0 &= 303990\cdot 10^{-5},\\
b_1 &= 203285\cdot 10^{-7}, & e_1 &= -224601\cdot 10^{-7},\\
b_2 &= -150627\cdot 10^{-9}, & e_2 &= 770600\cdot 10^{-10},\\
b_3 &= 625767\cdot 10^{-12}, & e_3 &= 687200\cdot 10^{-12},\\
b_4 &= -134933\cdot 10^{-14}, & e_4 &= -594400\cdot 10^{-14},\\
b_5 &= 113867\cdot 10^{-17}, & e_5 &= 144107\cdot 10^{-16}.
\end{aligned}$$

TABLE 1. Values of the Coefficients B_p (cm^3/g), E_p (cm^3/g)^4, and H_p (cm^3/g)^7 Calculated on the Basis of Polynomials (6), (7), (8), and (9)

t, °C	Water		Toluene	
	B_p	$-E_p$	$-B_p$	H_p
0	1,7114	1,7124	1,4655	3,0399
25	1,3961	1,4058	1,0422	2,5351
50	1,0804	1,1200	0,7555	2,1628
75	0,8018	0,8665	0,5641	1,9249
100	0,5792	0,6582	0,4367	1,8014
125	0,4195	0,5062	0,3505	1,7672
150	0,3233	0,4194	0,2900	1,8092
175	0,2890	0,4077	0,2459	1,9427
200	0,3155	0,4881	0,2133	2,2289
225	0,4036	0,6943	0,1908	2,7913
250	0,5560	1,0891	0,1788	3,8328
275	0,7756	1,7802	—	—
300	1,0629	2,9393	—	—

TABLE 2. Values of the Prandtl Number for Toluene Calculated from Eq. (1) (first line) and from Eq. (5) (second line)

p, bar	Pr at the specified temperature, °C									
	25	50	75	100	125	150	175	200	225	250
p_s	7,42	6,15	5,40	4,86	4,41	4,12	3,84	3,58	3,31	3,10
	7,42	6,15	5,40	4,86	4,41	4,12	3,83	3,58	3,30	3,10
25	7,54	6,21	5,44	4,91	4,48	4,18	3,88	3,61	3,35	3,13
	7,55	6,23	5,46	4,91	4,46	4,14	3,86	3,61	3,34	3,12
50	7,63	6,27	5,50	4,96	4,52	4,21	3,92	3,66	3,40	3,15
	7,64	6,29	5,51	4,96	4,50	4,18	3,90	3,65	3,38	3,18
75	7,72	6,33	5,55	4,99	4,58	4,25	3,96	3,71	3,44	3,19
	7,74	6,35	5,56	5,00	4,54	4,22	3,94	3,69	3,43	3,24
100	7,82	6,40	5,59	5,04	4,62	4,30	4,01	3,75	3,50	3,25
	7,82	6,41	5,61	5,04	4,57	4,25	3,97	3,72	3,47	3,29
125	7,91	6,47	5,66	5,08	4,66	4,32	4,05	3,81	3,57	3,33
	7,91	6,47	5,66	5,08	4,61	4,28	4,00	3,76	3,51	3,34
150	8,00	6,53	5,69	5,12	4,70	4,38	4,09	3,85	3,62	3,37
	8,01	6,54	5,71	5,13	4,64	4,32	4,04	3,79	3,55	3,39
175	8,09	6,60	5,75	5,17	4,74	4,40	4,14	3,89	3,67	3,43
	8,09	6,59	5,75	5,17	4,68	4,35	4,07	3,82	3,58	3,43
200	8,18	6,65	5,80	5,21	4,77	4,44	4,18	3,94	3,73	3,47
	8,18	6,66	5,80	5,21	4,71	4,38	4,10	3,85	3,62	3,48
225	8,26	6,74	5,85	5,25	4,81	4,46	4,20	3,98	3,77	3,52
	8,28	6,72	5,84	5,25	4,75	4,41	4,13	3,89	3,66	3,52
250	8,35	6,78	5,89	5,28	4,85	4,49	4,23	4,01	3,80	3,57
	8,36	6,78	5,90	5,28	4,73	4,44	4,16	3,92	3,69	3,57

TABLE 3. Values of the Coefficients $m_1(t)$, $n_1(t)$ (μ/W) and $m_2(t)$, $n_2(t)$ (kg·deg K/J) in Eqs. (12) and (16)

t, °C	Water				Toluene			
	$m_1(t)$	$n_1(t)$	$m_2(t)$	$n_2(t)$	$m_1(t)$	$n_1(t)$	$m_2(t)$	$n_2(t)$
0	38,04	—44,03	—86,84	56965	—	—	—	—
25	—	—	—	—	—6,02	103,80	3,07	7708,3
50	7,82	—6,66	28,38	—45676	—2,00	66,58	3,11	7227,3
75	—	—	—	—	—0,14	47,91	3,08	6950,8
100	3,02	—1,90	3,54	—64732	0,58	39,13	2,97	6973,1
125	—	—	—	—	0,55	37,35	2,73	7547,3
150	1,60	—0,67	1,51	—2071,8	1,08	30,94	2,74	7342,0
175	—	—	—	—	1,97	30,88	2,58	7940,2
200	1,30	—0,60	1,19	—2125,9	0,72	32,39	2,42	8754,7
225	—	—	—	—	0,35	35,00	2,2	9761,7
250	1,52	—1,10	1,42	—5396,4	0,62	30,69	2,23	9352,6
275	—	—	—	—	—	—	—	—
300	2,21	—2,32	2,62	—18434	—	—	—	—

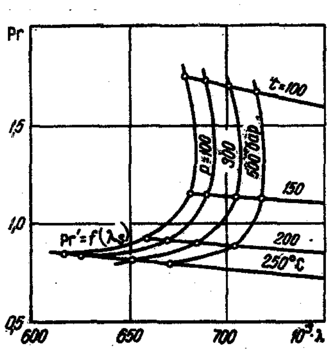

Fig. 1

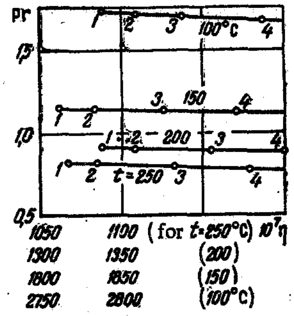

Fig. 2

Fig. 1. The function Pr = f(λ) for water.

Fig. 2. The function Pr = $\varphi(\eta)$ for water. 1) Liquid in saturated state; 2) p = 100 bars; 3) 300; 4) 500.

The numerical values of these coefficients are shown as functions of the temperature in Table 1.

Table 2 shows the values of Pr for toluene calculated from Eqs. (1) and (5). Comparison of the Prandtl numbers calculated from Eq. (1) on the basis of the experimental data for water and toluene with the result calculated from Eqs. (4) and (5) shows that the proposed method gives accurate values of the Prandtl number for the state parameters under consideration here. Since the equations of state in (2) and (3) describe the specific volume of water up to 1000 bars and that of toluene up to 500 bars, we must assume that Eqs. (4) and (5) should also give good results for Pr up to these pressures.

We have found that the equations for the transport coefficients can also be written in a form analogous to an equation of state [8-10] and that, furthermore, there is a linear relation between these transport coefficients [11].

Using these conclusions and Eqs. (4) and (5), we find the following relations between various combinations of the thermal conductivity, the viscosity, and the Prandtl number:

$$\frac{\frac{\lambda}{\lambda_s'}-1}{E_\lambda}=\frac{\frac{\eta}{\eta_s'}-1}{E_\eta}=\frac{\frac{\mathrm{Pr}}{\mathrm{Pr}'}-1}{E_p} \tag{10}$$

for water and

$$\frac{\frac{\lambda}{\lambda_s'}-1}{H_\lambda}=\frac{\frac{\eta}{\eta_s'}-1}{H_\eta}=\frac{\frac{\mathrm{Pr}}{\mathrm{Pr}'}-1}{H_p} \tag{11}$$

for toluene.

Now using (4') and (5'), we can find a common equation giving the linear relations between the Prandtl number and the transport properties for these two liquids:

$$\mathrm{Pr}=\mathrm{Pr}'\left(1-\frac{B_p}{B_\lambda}\right)+\frac{\mathrm{Pr}'}{\lambda_s'}\cdot\frac{B_p}{B_\lambda}\cdot\lambda$$

or

$$\mathrm{Pr}=m_1(t)+n_1(t)\lambda, \tag{12}$$

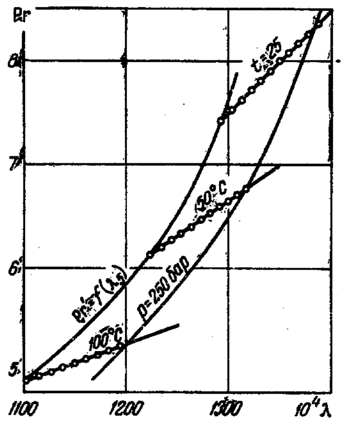

Fig. 3

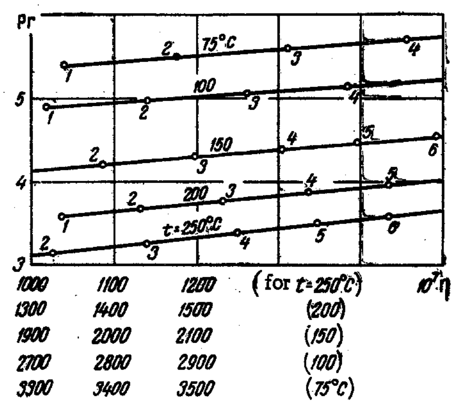

Fig. 4

Fig. 3. The function Pr = f(λ) for toluene.

Fig. 4. The function Pr = $\varphi(\eta)$ for toluene. 1) Liquid in saturated state; 2) p = 50 bars; 3) 100; 4) 150; 5) 200; 6) 250.

where

$$m_1(t) = \mathrm{Pr}'\left(1 - \frac{B_p}{B_\lambda}\right), \tag{13}$$

$$n_1(t) = \frac{\mathrm{Pr}'}{\lambda_s'} \cdot \frac{B_p}{B_\lambda} \tag{14}$$

and

$$\mathrm{Pr} = \mathrm{Pr}'\left(1 - \frac{B_p}{B_\eta}\right) + \frac{\mathrm{Pr}'}{\eta_s'} \cdot \frac{B_p}{B_\eta} \cdot \eta, \tag{15}$$

or

$$\mathrm{Pr} = m_2(t) + n_2(t)\,\eta, \tag{16}$$

where

$$m_2(t) = \mathrm{Pr}'\left(1 - \frac{B_p}{B_\eta}\right), \tag{17}$$

$$n_2(t) = \frac{\mathrm{Pr}'}{\eta_s'} \cdot \frac{B_p}{B_\eta}. \tag{18}$$

Figures 1-4 illustrate the linearity of the functions Pr = f(λ) and Pr = $\varphi(\eta)$ for water and toluene.

The numerical values of $m_1(t)$, $n_1(t)$ and $m_2(t)$, $n_2(t)$ in Eqs. (12) and (16), calculated on the basis of two points, are shown in Table 3.

In conclusion, we should point out that the linear relations which we found between the Prandtl number, on the one hand, and the transport properties, on the other, for water and toluene are also valid for other liquids, if these liquids obey the theoretically justified "particular case of the equation of state in virial form," incorporating various combinations of only two virial coefficients.

NOTATION

p, pressure, bars; v, specific volume, cm^3/g; ρ, density, g/cm^3; ρ_s, density of saturated liquid, g/cm^3; ρ_0, density of liquid for pv/RT = 1, g/cm^3; R, universal gas constant, (cm^3/g)(bar/deg K); T, absolute temperature,

°K; t, temperature, °C; B, B_p, B_λ, B_η, (cm^3/g), E, E_p, E_λ, E_η [($cm^3/g)^4$], H, H_p, H_λ, H_η, [($cm^3/g)^7$], coefficients of the temperature functions; Pr, Prandtl number; η, dynamic viscosity, $N \cdot sec/m^2$; λ, thermal conductivity, W/(m·deg K); c_p, specific heat at constant pressure, J/(kg·deg K); Pr', η'_s, λ'_s, c'_{ps}, properties in the saturated state.

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THERMAL CONDUCTIVITY OF POLYMETHYLPHENYLSILOXANES AT HIGH PRESSURES

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Results are presented of an experimental investigation of the thermal conductivity of polymethylphenylsiloxanes in the 20-200°C temperature range and up to 200-MN/m^2 pressure range. An equation is proposed to compute the heat conduction in the temperature and pressure ranges investigated.

Results of an experimental investigation of the thermal conductivity (λ) of polymethylphenylsiloxanes (PFMS) and polymethylsiloxanes (PMS) at atmospheric pressure are presented in papers [1-3].

Results of an investigation of the thermal conductivity of PFMS-4 and PFMS-2/51 at pressures up to 200 MN/m^2 and in the 20-200°C temperature range are presented in this paper. The thermal conductivity was measured in an apparatus whose measuring cell operates according to the method of coaxial cylinders with flat

TABLE 1. Characteristics of Polymethylphenylsiloxane Fluids

Polymer	ρ_4^{20}	ρ_{20}, kg/m^3	n_D^{20}	$\nu_{20} \cdot 10^6$, m^2/sec	M	N	λ_{20}, W/m·deg	$\left(\frac{\partial\lambda}{\partial t}\right)_p^{\overline{av}} \cdot 10^4$ atm
PFMS-4	1,1015	1094,0	1,5320	658,7	1328	9	0,1456	5,10
PFMS-2/51	1,0196	1012,0	1,4982	16,93	570	3	0.1278	5,17

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